Characterising the slow extraction frequency response

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Overview

A dedicated study performed at CERN SPS on the frequency transfer of slow extraction (current $\longrightarrow$ spill):

- $\longrightarrow$ Development of different models of the process
- $\longrightarrow$ Simulation and characterization
- $\longrightarrow$ Experimental measurements

Main goals:

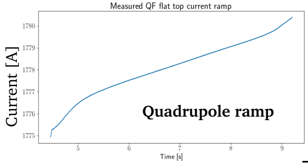
- Better understanding and use of the freq. transf. process
- Identify possible improvements in the ripple suppression and spill quality

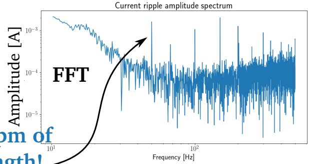
References:

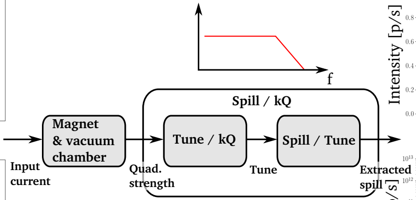
Phys. Rev. Accel. Beams 24, 083501 (2021)

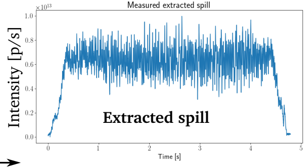
M.Pari, PhD Thesis, University of Padova
http://paduaresearch.cab.unipd.it/13202

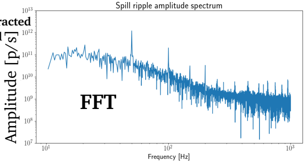
Problem modeled focusing on the transfer function formalism:
block diagram of different elements acting in frequency domain











Order of ppm of quad. strength!

Perturbation approach:

$$i + \Delta i \longrightarrow s + \Delta s$$

ripples

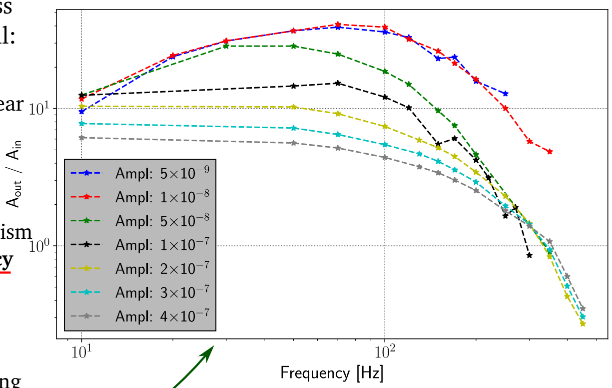
SPS slow extraction:
horizontal momentum extraction

→

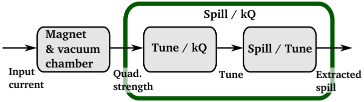
For this study: QF current as input,
(dominant contribution to extraction)

Slow extraction process known to behave as low-pass filter between input quad. current and extracted spill:

- Evolution of particles in phase space in strong non-linear conditions: "low level" modeling is rather complex
- High-level approach based on transfer function formalism developed for this study: **injection of single-frequency sinusoid to characterize system response**
- Response of **full slow extraction** process obtained using MADX simulation of SPS slow extraction

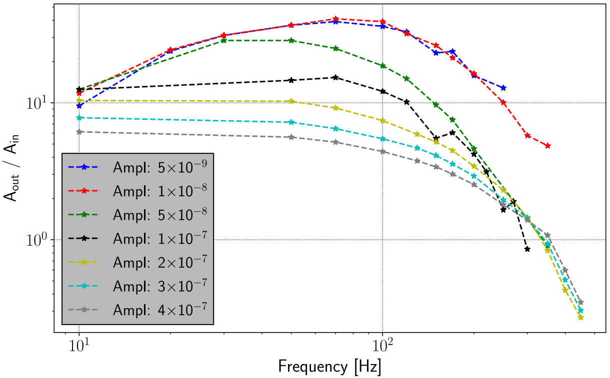


Results



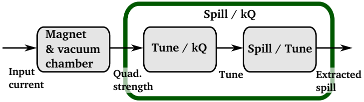
Remarks:

- The problem is non-linear for high ripple amplitudes (amplitude-dependent), becomes linear below a certain threshold (blue red)
- Transfer function pole at ∼ 100 Hz
- Transfer function zero only for small injected amplitudes



The results of this characterization in terms of transfer function give useful insight on the problem

(next)



Linearity vs non linearity

Simple spill expression for mom. extr. in low freq. (instantaneous) approximation can already explain non linear transition:

$$\frac{dN(t)}{dt} = \begin{cases} C\frac{dQ(t)}{dt} & \text{if } Q(t) > Q(\tau)|_{\tau\in[t_i,t[} \\ 0 & \text{otherwise} \end{cases}$$

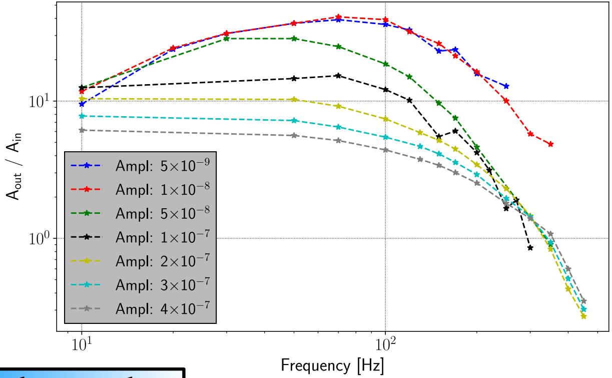
(but no info on low pass filter effect in this expr.)

Monotonic tune ramp → **Linear problem**

Restricting the case to linear ramp + sinusoidal perturbation: simple linearity condition

$$A_Q 2\pi f \le \left|\frac{\Delta Q}{\Delta t}\right|$$

important role of tune ramp slope



using the SPS values for a 50 Hz ripple:

$$2.4\times10^{-8}\ \mathrm{m}^{-2}$$

equivalent to ~1.6 ppm q.str.

good agreement with the threshold from simulation:

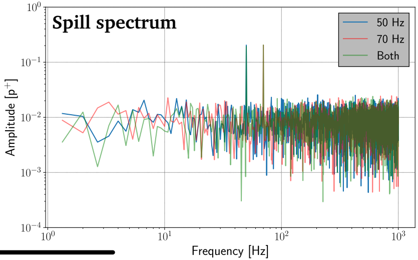
$$2.5\times10^{-8}\ \mathrm{m}^{-2}$$

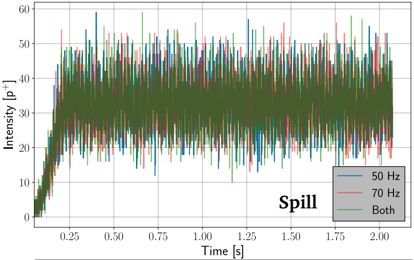
Linearity vs non linearity

A useful visual example: **simulated injection of 50 and 70 Hz sinusoidal ripples (below cut-off freq.)**

Small amplitude
(below linear threshold)

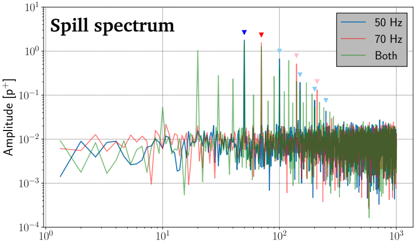
- Same inj. freq. on the spill
- Superposition principle holds
- Continuous spill

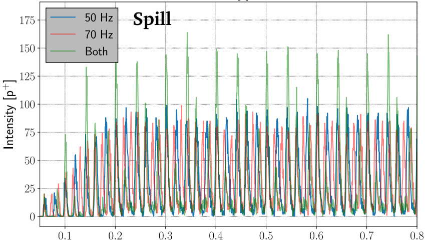




High amplitude
(above linear threshold)

- Appearance of harmonics
- Superposition principle broken
- Spill split in pulses



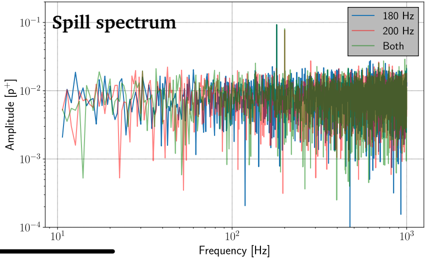


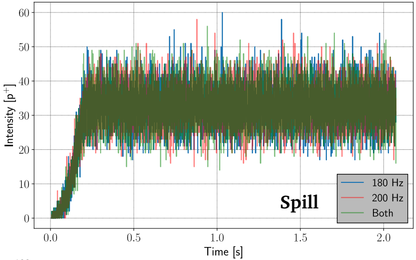
Linearity vs non linearity vs low pass

A useful visual example: **simulated injection of 180 and 200 Hz sinusoidal ripples (above cut-off freq. : action of low pass filter effect)**

Small amplitude
(below linear threshold)

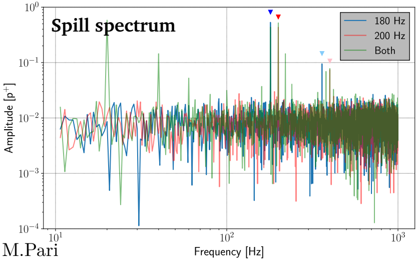
- Same inj. freq. on the spill: **reduced amplitude**
- Superposition principle holds
- Continuous spill: **improved**

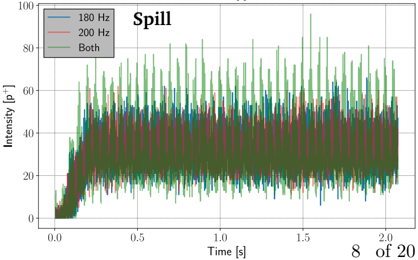




High amplitude
(above linear threshold)

- Non linearity but
- **Low pass filter is evident!**
  Less harmonics & < ampl
- **Spill not fully pulsed**





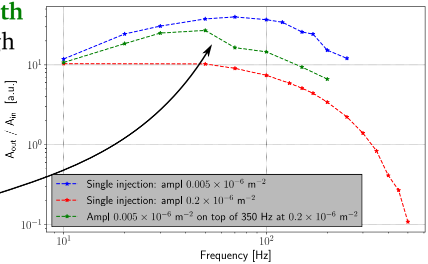
Dominant ripple injection

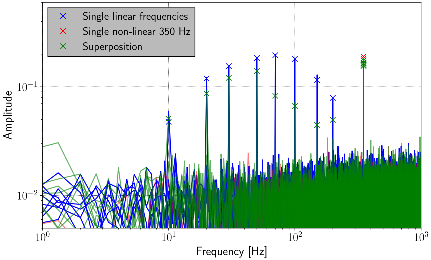
Application: use the transfer function formalism to observe the method developed at GSI SIS-18 [1,2] of a dominant ripple injection

Injection of LINEAR ripples **with** and **without** NON LINEAR high freq. (above cut-off) ripple.

Result:

- **Linear ripples suppressed when non lin. present (up to ∼1/2)**
- **Breaking linearity necessary condition for this to happen**



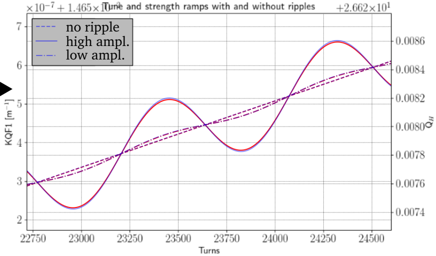


The problem could be studied parametrically for the best suppression, but experiments @ CERN SPS sensible to high freq. ripples: need dedicated investigation w/ experiments to continue

[1] Phys. Rev. Applied 13, 044076 (2020)
[2] SXW2019 - https://indico.fnal.gov/event/20260/contributions/56672/

The full slow extraction block can be decomposed and modeled as follows:

Tune/kQ block: approximated with a linear law

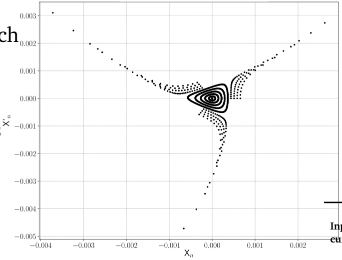


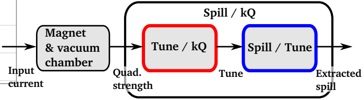
Spill/Tune block:

- Custom Henon-map based simulation model developed

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{n+1} = \mathbf{R}\left(2\pi Q_n\right) \begin{pmatrix} x \\ x' + V_{\mathrm{ss}}\, x^2 \end{pmatrix}_n$$

- Main parameters of the SPS used to match it to MADX
- Simplified model: important advantage in simulation times (up to $\sim 1/100$)
- Same sinusoidal ripple injection procedure for response characterization



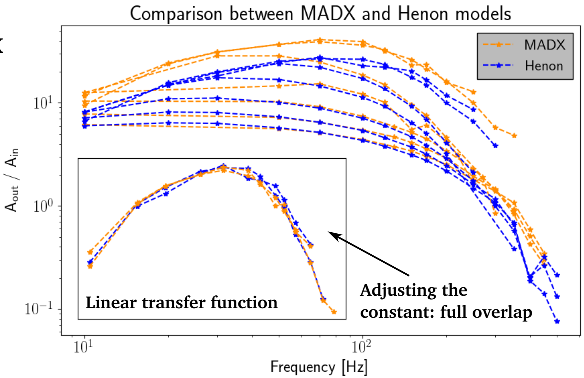


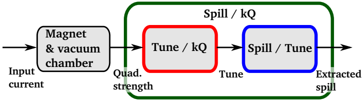
Development of a model

The transfer functions from the complete MADX model and the Henon one can be scaled to be fully compatible:

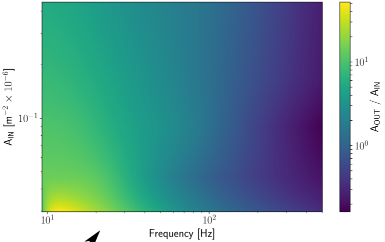
⟶ In particular, the **linear** transfer functions (generic & amplitude independent) show good agreement

⟶ Henon-map model captures the essence of the process



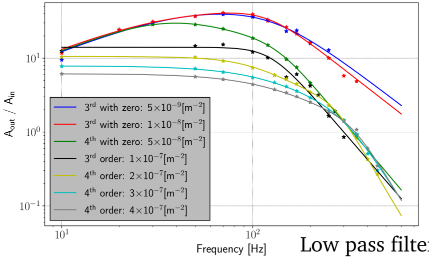


Semi-analytic expression of frequency response obtained by modeling each fixed-amplitude response with analytic low-pass filter function



E.g. linear response univocally determined as 3rd order low pass filter with a zero:

$$F_{3z}(\omega) = \frac{K \cdot \omega}{\sqrt{\left(1+\left(\frac{\omega}{\omega_c}\right)^2\right)\left(\left(1-\left(\frac{\omega}{\omega_c}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_c}\right)^2\right)}}$$



Low pass filter order grows with amplitude (non-linear region)

- **Can be used for instantaneous prediction of measured current on the spill (w/ some caveats from non linearity)**

Experimental measurements

Dedicated ripple injection measurements performed at the SPS:

$\longrightarrow$ Injected single frequency sinusoidal ripples w/ high amplitudes (10s of ppm): non linear regime

$\longrightarrow$ Ripple = voltage signal injected on power converters of focusing quads

$\longrightarrow$ Both reference and real current sampled at 1 kHz

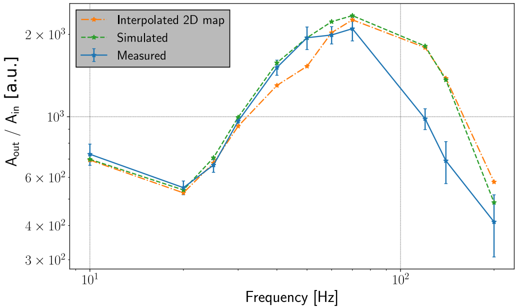
$\longrightarrow$ Spill signal sampled with SEM (BSI) at 2 kHz

$\longrightarrow$ Extracted intensity $\sim 10^{11}$ protons: two orders of magnitudes lower than nominal (due to external conditions)

Experimental measurements

These measurements are used to validate the MADX simulations and the semi-analytic 2D map: high amplitudes - non linear regime

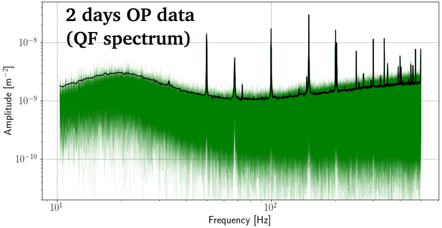
- ✓ Good agreement with simulations for most of injected frequencies.
- ✓ Good agreement between fully simulated points and semi-analytic extrapolation
  - ⟶ Only 5 transfer function used for the interpolation: if needed, precision can be further improved
- ✓ No evidence of hardware (vacuum chamber and magnet) effects at the observed frequencies (expected at a few kHz)



Operational data

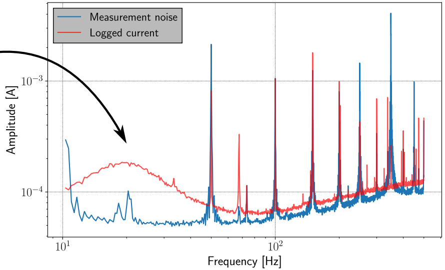
Operational data (physics runs) makes up for another good validation of the model:

- Same logged quantities as in dedicated injection meas. (spill, currents)
- Full intensity extraction ($\sim 3 \cdot 10^{13}$ p+): better frequency analysis
- Low amplitude ripples ($\lesssim$ ppm) for validation of the linear transfer function



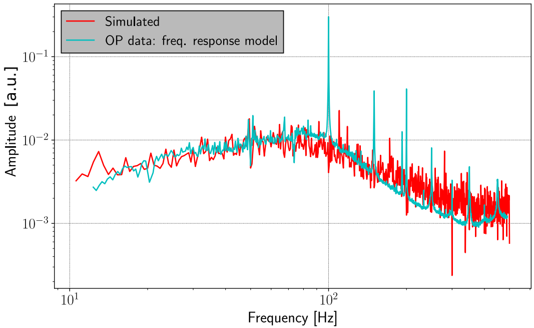
But ... is the measured ripple current reliable?

- No! Same order ripples from the current measurement chain (magnets OFF)
- Still, this measurement noise can be removed if restricting to the continuous spectrum



Operational data

- ✓ **Removed continuous noise floor using theoretical transfer function of measurement chain (*) (50 Hz harmonics not reliable)**
- ✓ **Used MADX model to simulated continuous transfer function from input noise**
- ✓ **Good agreement with simulations**



(discrete peaks artifact of analysis)

(*) Thanks to M. Cerquiera, M. Martino for the help

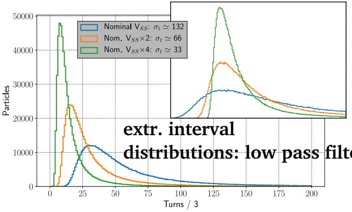
Fast prediction and scans

The Henon-map model showed good agreement w/ full MADX one

use it to scan main main SPS extraction parameters and look for possible improvements wrt nominal configuration

Scan parameters are virtual sextupole strength (V_{SS}) and chromaticity (ξ): both critical SX params

Virtual Sextupole Strength:



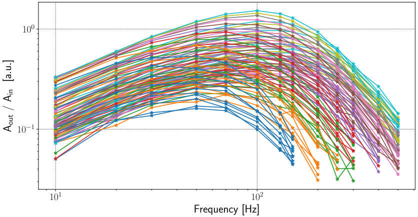
extr. interval distributions: low pass filter!

Chromaticity:

For a linear ramp + sin. ripple, the relative (low-freq) ripple amplitude on the spill is $\propto 1/\xi$

Plus linearity condition favors high chroma (=higher Q slope)

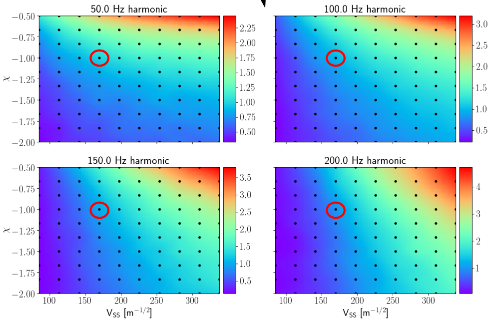
10x10 chroma Vss grid scan: linear transfer functions



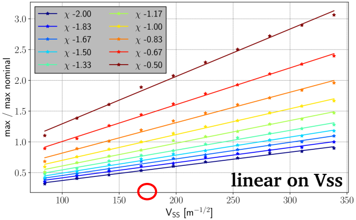
Investigation of different scenarios

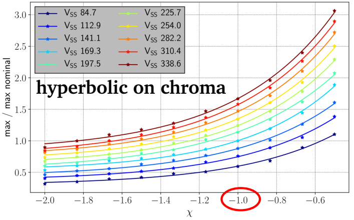
Suppression of ∼1/2 at 50 Hz within reach for lower sextupole strength and higher chroma

= nominal SPS



Maximum

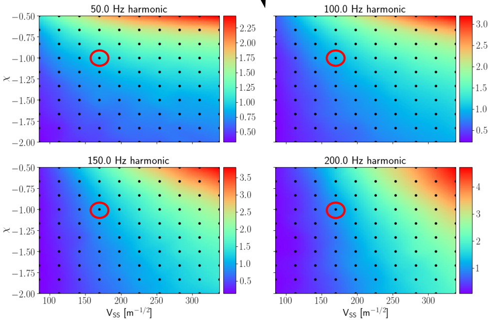




Main parameters of transfer function (e.g. pole, max) modeled as a function of the chroma and sext. strength:

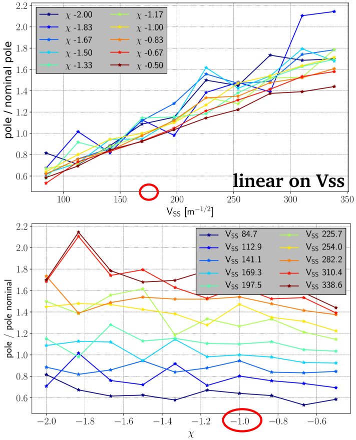
⟶ **Dependence can be approximated by analytic functions, allowing to develop analytical model**

Suppression of ~1/2 at 50 Hz within reach for lower sextupole strength and higher chroma



= nominal SPS

Pole



Main parameters of transfer function (e.g. pole, max) modeled as a function of the chroma and sext. strength:

⟶ **Dependence can be approximated by analytic functions, allowing to develop analytical model**

Conclusions and next steps

✓ Characterization of SPS slow extraction frequency response w/ full MADX sim & custom Henon-map model: agreement

✓ Process described by linear transfer function for small signals, semi-analytical model can be built for non linear ripples.

✓ Measurements @SPS in good agreement with developed models: injected non linear ripples & OP data

✓ Fast Henon-map simulation to scan main extraction parameters, allowing to identify possible improved configurations

✓ The study is summarized in **Phys. Rev. Accel. Beams 24, 083501 (2021)**

Next:

→ Further study & testing of the potential improved configurations

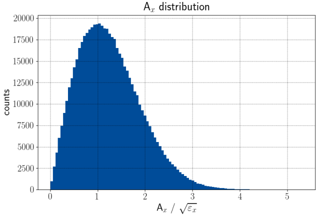
→ Address the issues for a precise measurement of the operational 50 Hz harmonics and noise spectrum

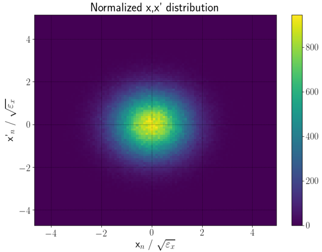
Thank you for your attention

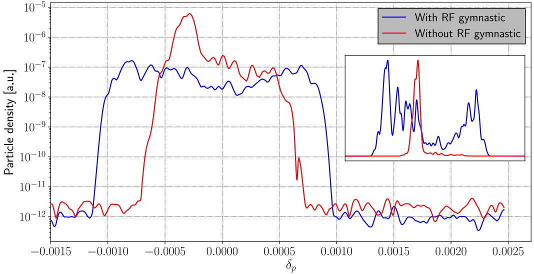
Backup

SPS parameters

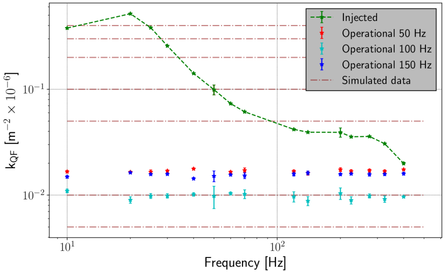
Parameter	Value
Momentum	400 GeV/c
Emittance	1.9×10^{-8} m
One turn time	23 µs
Chromaticity	-26.67
Resonant tune	$26.6\bar{6}$
Start of tune ramp	26.62
End of tune ramp	26.72
Flat top duration	4.8 s
Momentum range (δ_p)	3×10^{-3}
Virtual sextupole strength	$169.3\ \mathrm{m}^{-1/2}$
Closed orbit bump (max)	48 mm
Electrostatic septum pos.	68 mm

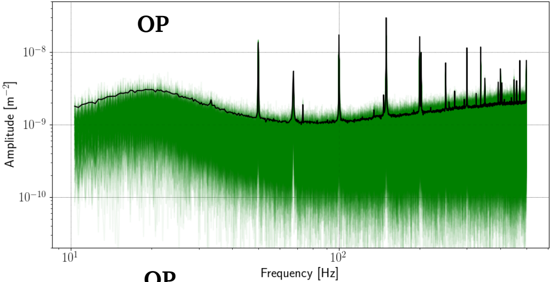


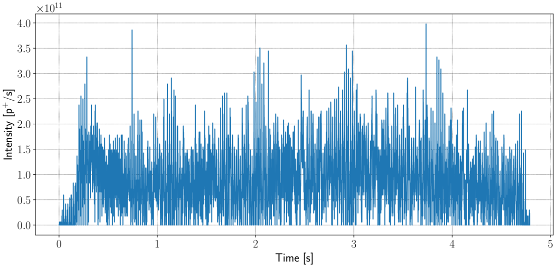


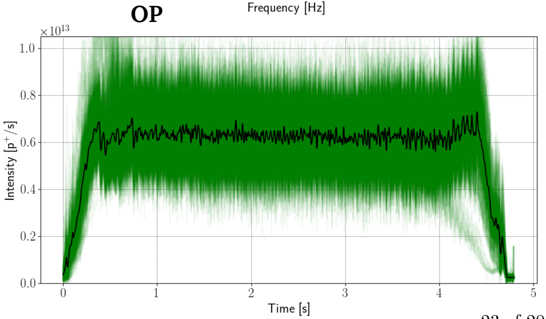


Freq. response: measurements









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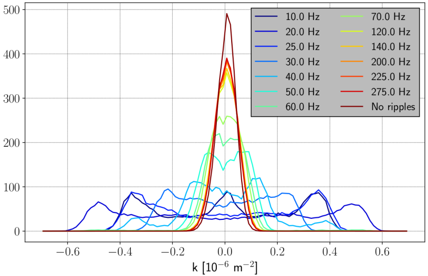

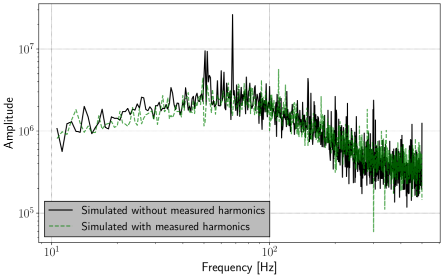

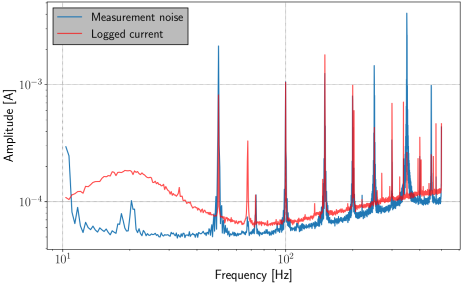

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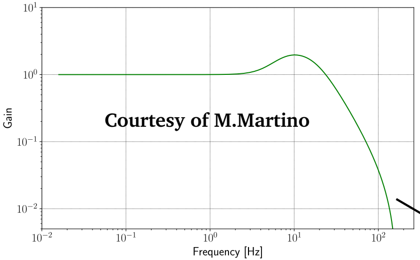
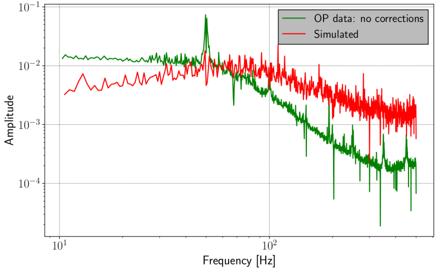
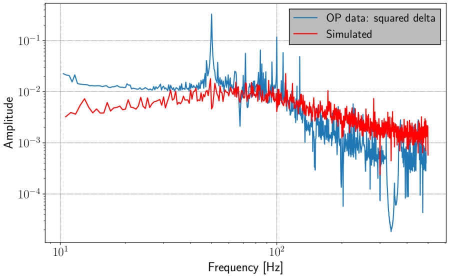
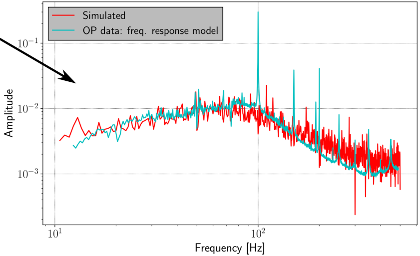
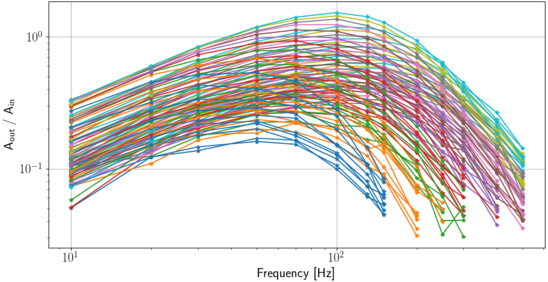




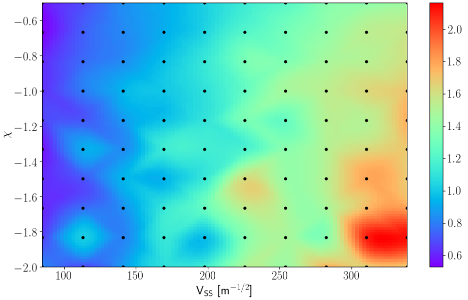




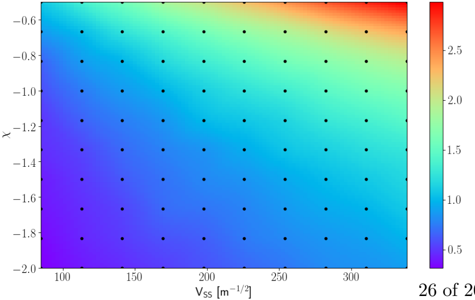
Freq. response: Henon map model



Pole



Max

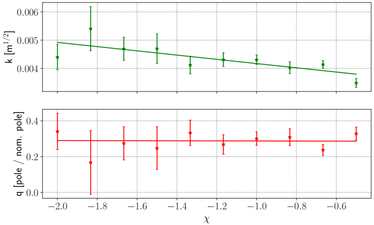


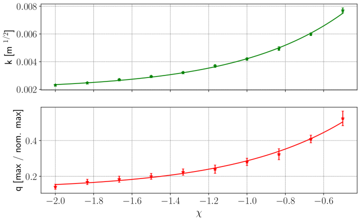
Freq. response: Henon map model

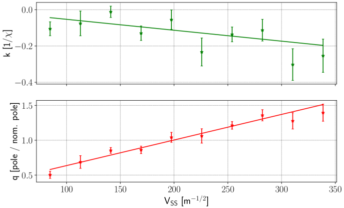
$$m(V_{\mathrm{ss}})\Big|_{\chi} = k \cdot V_{\mathrm{ss}} + q$$

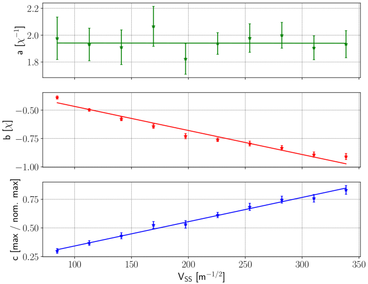
$$m(\chi)\Big|_{V_{\mathrm{ss}}} = e^{a\cdot(\chi - b)} + c$$

$$p(V_{ss}, \chi) \simeq k \cdot V_{ss} + q$$









Freq. response: other accelerators

Bonus of Henon map model: readily applied to different accelerators

Parameter	CERN-SPS	MedAustron
Momentum	400 GeV/c	$\leqslant$ 250 MeV/c
One turn time	23 µs	420 ns
Chromaticity	26.67	4
Total tune sweep	0.1	0.02
Flat top duration	4.8 s	9 s
Momentum range (δ_p)	3×10^{-3}	5×10^{-3}
Virtual sextupole strength	169.3 $\mathrm{m}^{-1/2}$	29.8 $\mathrm{m}^{-1/2}$
Emittance	1.9×10^{-8} m	6.6×10^{-7} m

Factor 50 between the poles

Measurements taken at MedAustron
by P.Arrutia M.Fraser M.Pivi et al:
to be continued in the future